

this point on they devote themselves almost exclusively to hyperbolic plane geometry in general, and Lobachevsky parallels in particular. It is needful to show a number of things about parallels, notably that if l is one of the parallels to m , through a point P , it is one of the parallels through every other point on itself. Much attention is given to the three types of pencils of lines, those through a point, those all parallel to one another, and those all perpendicular to a given line. The main object is to develop the formula for the parallel angle, and so the method for drawing parallels. The leading idea is taken from Engel, and consists in establishing a correspondence between a right triangle, and a quadrilateral with three right angles.

The hyperbolic circle first appears on page 62 and is defined as the locus of the reflection of a point in the lines of a pencil. When the pencil consists in concurrent lines this gives the proper circle, when it is a parallel pencil, this is the queer circle called a horocycle, when the pencil is made up of lines perpendicular to a given line this should give the equidistant curve, i.e. the locus of all points at the same distance as the given point from the given line. It will, however, only give one half of that locus. Next comes an elaborate study of the concept of area, and of trigonometry. Twice we find the functional equation

$$f(x) + f(y) = f(x+y).$$

We are told that it is "geometrically evident" that this function is continuous, hence the solution is $f(x) = rx$. But if we were content with those things which are "geometrically evident" we should not bother with non-euclidean geometry at all, for it is sufficiently evident geometrically that if Euclid's parallel axiom is not true, the amount of its falsity is not enough to disturb the equanimity of sensible people. It is not thus that real mathematics is made. The subject of three-dimensional geometry is covered in eight pages at the end. Of such interesting subjects as Clifford parallels, not a trace.

Such is the book. Interesting and clear, and impregnated with the spirit of the founders, but, at best, one of many, presenting no striking advances over others already available.

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The Principles of Thermodynamics. By George Birtwistle, Fellow of Pembroke College, Cambridge. Cambridge University Press, 1925. 163 pp.

The book is a careful presentation of the subject to students who are presupposed to have only a general knowledge of the physical science. For this reason the relation of statistical mechanics to thermodynamics, and the connection between magnetism and temperature, for which a special knowledge of dynamical or physical theory is required, are not included. The third law of thermodynamics is likewise not considered. The work is prefaced in its first chapter by concise historical and descriptive statements relating to the theory of heat, temperature measurement, laws of Boyle, Charles and Avogadro, and the conception of a perfect gas. The