A QUALITATIVE DEFINITION OF THE POTENTIAL FUNCTIONS*

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1. *Introduction.* In this paper we aim to set up postulates completely characterizing the potential functions, which do not involve derivatives or integrals, and are thus of a more qualitative nature than the definitions previously given. Incidentally, we may interpret all of our postulates as statements of properties of such physical quantities as give rise to potential functions, and when so interpreted, we see from physical grounds that they hold for the quantities in question. They thus furnish a means of going directly from certain physical problems to the potential functions, without the use of Laplace's equation. Our results will be stated in full only for potential functions of two and three variables, although they may evidently be extended to the n-dimensional case.

2. *Postulates for Two Dimensions.* Consider a class of functions of two variables, x and y, thought of for convenience as Cartesian coordinates, and let each function have associated with it a region R of the plane. Our assumptions are:

(1) Each function is continuous in both variables at all interior points of its region R.

(2) If R_1 and R_2, the regions for two functions of the class $f_1(x, y)$ and $f_2(x, y)$, have a region R_3 in common, then any linear combination of these functions, such as $Af_1(x, y) + Bf_2(x, y)$, is a function of the class, whose region contains all the points of R_3.

(3) If an orthogonal transformation of the variables (i. e., a change of Cartesian axes) converts the function $f(x, y)$ with region R into $F(x', y')$, this latter function is a member of the class, its region being the region R expressed in the new variables.

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