COMPLETE SETS OF REPRESENTATIONS OF TWO-ELEMENT ALGEBRAS*

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1. *Introduction.* Any algebra consists of one or more classes of elements and one or more operations or relations among the elements. An operation $\oplus$ of a class K of two elements a, b is given by a $\oplus$-table of the form

$\oplus$	a	b
a	c_1	c_2
b	c_3	c_4

where the c's all belong to K, or do not all belong to K, according as the operation is K-closing or not. A relation R in K is given by an R-table of the form

R	a	b
a	$\pm$	$\pm$
b	$\pm$	$\pm$

where the sign $+$ indicates that aRb holds, the sign $-$ that aRb does not hold. The object of this paper is to give convenient representations of these $\oplus$-tables and R-tables, hence of all two-element algebras, and to point out the usefulness of these representations in connection with fundamental questions relating to postulate-sets.

2. *Representation of Class-closing Operations.* There are 2^4 operations $\oplus$ possible in a class of two elements when the condition of closure is satisfied. The following theorem gives us two sets of representations of these operations, one arithmetic and one boolean. In the boolean representation the symbols 0, 1, a', $a+b$, ab denote respectively

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(1) *An arithmetic representation of boolean logic,* (2) *Arithmetic independence systems for the Whitehead-Huntington postulates for boolean algebras,* (3) *A boolean representation of a number field.*