# APPLICABILITY WITH PRESERVATION OF BOTH CURVATURES*

BY W. C. GRAUSTEIN

1. *Introduction.* In his celebrated memoir of 1867, Bonnet† proved that there is in general no surface applicable to a given surface with preservation of both the total and mean curvatures, but that in certain exceptional cases there exists a unique surface or a one-parameter family. The primary purpose of this note is to exhibit conditions of simple form characterizing the second of these exceptional cases. The method of treatment applies equally well to the general case.

2. *Necessary and Sufficient Condition.* It is well known that a necessary condition that a surface $S$ admit $\infty^1$ surfaces applicable to it with preservation of both curvatures is that the surface $S$ be isometric.‡ Accordingly, we can assume that $S$ is an isometric surface, referred to its lines of curvature, and that the parameters are isometric. The linear element of $S$ is then of the form

$$ds^2 = \lambda(du^2 + dv^2), \tag{1}$$

* Presented to the American Mathematical Society, September 7, 1923.

† *Mémoire sur la théorie des surfaces applicables sur une surface donnée,* JOURNAL DE L'ÉCOLE POLYTECHNIQUE, vol. 42 (1867), pp. 72 et seq.

‡ This fact appears to have been discovered first by Raffy, *Sur une classe nouvelle de surfaces isothermiques et sur les surfaces déformables sans alteration des courbures principales,* BULLETIN DE LA SOCIÉTÉ DE FRANCE, vol. 21 (1893), pp. 70–72. His proof, however, is faulty, in that he bases part of it on the incorrect statement, made by Caronnet, that every $W$-surface applicable to a surface of revolution is isometric. If that statement were correct, every helecoidal surface would be isometric, which is not the case. A valid proof is given by Hazzidakis, *Biegung mit Erhaltung der Hauptkrümmungsradien,* JOURNAL FÜR MATHEMATIK vol. 117 (1897), p. 46.