INTEGRAL SOLUTIONS OF $x^2 - my^2 = zw$ *

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1. *Statement of the Theorem.* In this BULLETIN (vol. 27 (1920–1), p. 361) I stated that all integral solutions of the equation

$$x^2 - my^2 = zw$$

are obtained by multiplying the right members of

$$(1) \quad z = el^2 + 2flq + gq^2, \qquad w = en^2 - 2fnr + gr^2,$$

$$(2) \quad x = \pm (eln + fnq - flr - gqr), \qquad y = lr + nq,$$

by the same arbitrary integer, where e, f, g take only those sets of integral values (finite in number) for which the first form (1) is a reduced quadratic form having the same discriminant $4m$ as $x^2 - my^2$, so that

$$(3) \qquad f^2 - eg = m.$$

In other words, we employ a single form $el^2 + \cdots$ from each class of quadratic forms of discriminant $4m$. The number of such classes is therefore the number of sets of formulas (1), (2) giving all integral solutions of $x^2 - my^2 = zw$. If we permit the interchange of z and w, we need retain only a single sign in (2). For, if we replace l, q, n, r by $n, -r, -l, q$, respectively, we find that z and w are interchanged, y is unaltered, and x is replaced by $-x$.

In the paper cited, I was led to the above theorem by the theory of ideals, and I gave a proof when there is a single class of quadratic forms.† I stated that a simpler proof of the general theorem follows by composition. I have since found the following still simpler proof.

2. *A Simplification.* Since we may lay aside a common factor of x, y, z, w, consider $X^2 - mY^2 = ZW$, where X, Y, Z, W

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† A complete proof of the general theorem by means of ideals has been found recently by G. E. Wahlin.