AN INTRODUCTORY ACCOUNT OF THE ARITHMETICAL THEORY OF ALGEBRAIC NUMBERS AND ITS RECENT DEVELOPMENTS*

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1. *Introduction.* In dealing with the subject of my lecture, I might have considered it from a purely logical point of view, and developed it in all its beauty in this manner. I think, however, it will be of more interest to you if I introduce it from the historical standpoint. Its beginnings date from Euler, who attempted to prove Fermat's statement, that the only integer solutions of the equation $y^2 + 2 = x^3$ were $x = 3$, $y = \pm 5$, by putting $x = a^2 + 2b^2$ and taking

$$y + \sqrt{-2} = (a + b\sqrt{-2})^3.$$

By equating irrational parts, he found

$$1 = b(3a^2 - 2b^2),$$

whence $b = 1$, $a = \pm 1$; but it is neither obvious nor true in general that all the integer solutions can be found in this way, —one used by Euler and Lagrange for some related questions. Then, about 1800, much interest was shown in the so-called law of quadratic reciprocity, first rigorously proved by Gauss; namely, that if p and q are two odd positive primes

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}.$$

The symbol (p/q) denotes $+1$ or -1, according as the congruence $x^2 \equiv p \pmod{q}$ is possible or impossible, and then p is called a quadratic or non-quadratic residue respectively of q. With certain extensions, this law is really equivalent to a reduction formula enabling us to calculate the value of the symbol (p/q), and forms the foundation of the theory of numbers.

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