NOTE ON A CERTAIN TYPE OF RULED SURFACE *

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In the January, 1923, number of this BULLETIN, J. K. Whittemore discusses ruled surfaces having the property that any two secondary asymptotic lines cut equal segments from the rulings. It so happens that, in investigating the determination of a surface by the linear element of its spherical representation and its total and mean curvatures, the present writer was led to consider the same class of ruled surfaces, with the results which are set forth in this note. The method of attack differs from that of Whittemore and the facts obtained overlap only in the case of the characteristic property, namely, that the rulings are parallel to a plane and the parameter of distribution is constant.

Any two secondary asymptotic lines of a ruled surface cut equal segments from the rulings if and only if the surface, when referred to its asymptotic lines as the parametric curves, admits a representation of the usual form,

$$x_i = \xi_i(v) + u\eta_i(v), \qquad (i = 1, 2, 3), \tag{1}$$

where η_1, η_2, η_3 are the direction cosines of the ruling, η, and u is the algebraic distance along the ruling from the directrix, $\xi = \xi(v)$. Analytically, this condition amounts to demanding that $D'' = 0$. But HD'' is a quadratic polynomial in u, whose coefficients are functions of v alone. Thus the condition $D'' = 0$ gives rise to three equations, namely:

$$(\eta\xi'\xi'') = 0, \qquad (\eta\eta'\xi'') + (\eta\xi'\eta'') = 0, \qquad (\eta\eta'\eta'') = 0. \tag{2}$$

The vanishing of the last determinant is the condition that the director cone degenerate into a plane. The spherical indicatrix, $\eta_i = \eta_i(v)$, $(i = 1, 2, 3)$, is then a circle of unit radius, in particular, the circle in which the director plane cuts the unit sphere. If we choose as the parameter v the arc of this circle, it follows that $\eta_i'' = -\eta_i$, $(i = 1, 2, 3)$, and conditions (2) become

$$(\eta\xi'\xi'') = 0, \qquad (\eta\eta'\xi'') = 0. \tag{3}$$

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