TOTAL GEODESIC CURVATURE AND GEODESIC TORSION*

BY J. K. WHITTEMORE

In a paper on the gyroscope, presented to this Society April 23, 1921, Professor W. F. Osgood introduced the notion of the "bending" of a spherical curve. The "bending" is defined as the rate of turning, per unit length of arc of the curve, of the plane determined by the tangent to the curve and the normal to the surface. It is the purpose of this note to show that the bending of a curve on any surface is equal to

$$\sqrt{\frac{1}{\rho_g^2}+\frac{1}{\tau_g^2}},$$

where ρ_g and τ_g are the radii of geodesic curvature and geodesic torsion respectively. An expression, believed to be new, for the geodesic torsion of any curve of the surface is also derived. Since the rate of turning of the principal normal of a curve,

$$\sqrt{\frac{1}{\rho^2}+\frac{1}{\tau^2}},$$

where ρ and τ are the radii of curvature and torsion respectively, is called the total curvature of a curve,† it seems appropriate to replace the term "bending" by "total geodesic curvature."

Let Γ be any curve of a surface S and P be any point of Γ; let $\bar{\omega}$ be the angle measured from the positive direction of the principal normal to Γ at P to the positive direction of the normal to S at P, the angle being measured toward the positive binormal; let the direction cosines of the positive direction of the normal to S be X, Y, Z, and those of the positive directions of principal normal and binormal be respectively l, m, n and λ, μ, ν. Then the direction cosines of the normal to the plane of the tangent to Γ and the normal to S are

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† ENCYCLOPÄDIE DER MATHEMATISCHEN WISSENSCHAFTEN, III, 3, 1, p. 82.