AN UNCOUNTABLE, CLOSED, AND NON-DENSE POINT SET EACH OF WHOSE COMPLEMENTARY INTERVALS ABUTS ON ANOTHER ONE AT EACH OF ITS ENDS*

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On page 92 of the 1907 edition of Hobson's *The Theory of Functions of a Real Variable,* and again on page 113 of the second edition of the same treatise, there occurs the following statement: †

"*A non-dense closed set is enumerable if its complementary intervals are such that every one of them abuts on another one at each of its ends.*"

To prove this statement, Hobson lets G denote the non-dense closed set in question and argues, in part, as follows:

"In this case, all the points of G are either end-points of adjacent intervals, or limiting points, on both sides, of a sequence of such end-points; unless a ‡ or b ‡ be a limiting point, in which case it belongs to G. The end-points have the same cardinal number as the rational numbers, since the set of intervals is enumerable. Moreover the external § points form a finite set, or an enumerable set; because to each such external point there corresponds an enumerable set of end-points of which it is the limiting point and in this correspondence *any one end-point can correspond to at most two limiting points, one on each side of it.*"

Just what is the meaning of the above italicized statement? If P is an external point, then, by definition, there exists a

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† This statement will be called Proposition A.

‡ Here a and b apparently denote the end-points of some interval which contains the set G.

§ External points are defined by Hobson as points such as are not end-points of any contiguous interval but are limit points on both sides of such end-points.