Cours de Géométrie Analytique. By Georges Bouligand, with a preface by E. Cartan. Paris, Librairie Vuibert, 1919. vii + 421 pp.

This book constitutes a second course in the Cartesian geometry of the plane and of space, freely interspersed with essentials of modern geometry. After a concise development of rectangular and oblique coordinates and their application to straight lines and planes, the infinite and the imaginary elements are introduced. With the basis for projective geometry in complex space or for any of its sub-geometries thus developed, the author proceeds to consider some general questions. He first treats miscellaneous elementary properties of real curves and surfaces in real space. He then passes on to the discussion of projective and affine properties of algebraic curves and surfaces in complex space, developing the ideas of the order of a curve or a surface and that of one of its points, proving Bezout's theorem, and devoting some attention to asymptotes and to unicursal curves. After a brief discussion of loci comes the next major topic, that of transformations. Rigid motions, reflections, and transformations of similitude are discussed, followed by affine and projective transformations; cross ratio is introduced and the projective generation of conics and quadrics developed, involutions are given proper attention and Desargues' theorem is proved; finally comes the geometry of inversion.

An important phase of this chapter is the stress laid on the distinction between metric, affine, and projective geometries. Indeed, if the reviewer were to single out any one motive governing the plan of the book, it would be this motive. Latent but still decisive in the beginning, when once introduced it maintains predominance.

One further general concept, that of tangential coordinates, is developed, and the author then attacks the theory of curves and surfaces of the second order. With the general equation as the point of departure in each case, the usual questions of classification, normal forms, various geometric definitions (of the conics), rulings and circular sections (of the quadrics), tangential equations, poles and polars, diameters, etc., are discussed; a separate chapter treats geometrically the intersection of two quadrics.

A supplement of some sixty pages covers the geometrical applications of determinants, the classification of conics and quadrics on an analytic basis, the determination of conics and quadrics under given conditions, pencils of conics and quadrics, and an introduction to the theory of invariants.

The differential calculus is freely applied and a modicum of differential geometry developed. Certain subjects, whose detailed analytic treatments are regarded, perhaps unfortunately, by most American geometers as belonging to analysis, are carefully treated: the implicit function theorem in connection with a plane curve defined by an implicit equation, and a thorough analytic theory of envelopes in connection with line coordinates. On the other hand, the principle of duality is barely mentioned and, though polar reciprocation with respect to the sphere, $x^2 + y^2 + z^2 + 1 = 0$, is freely used, the theory of reciprocal polars in general is lightly passed over and that of general correlations not mentioned.

All in all, however, the author is to be congratulated on his choice of