

KINEMATICS IN A COMPLEX PLANE AND SOME GEOMETRIC APPLICATIONS *

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1. *Introduction.* From an elementary standpoint one is apt to consider geometry in a complex plane as an accessory of heuristic value for function theory. In a more advanced sense one recognizes the fundamental importance and intrinsic value of the geometric problem of partition of the complex plane by circular arcs in connection with the properties and classification of certain linear substitution groups, with the corresponding automorphic functions, and, in particular, with the theory of algebraic curves and their Riemann surfaces and uniformization. But even in a more elementary sense, the complex plane is the natural medium for the solution of certain specific geometric problems. As an example may be mentioned the "geometry of the polynomial," involving the theory of stelloid and circular curves and their focal properties.†

Also a number of problems in geometric kinematics may be solved conveniently in a complex plane, as has been shown by Koenigs,‡ Study,§ and others. In the present paper I shall show by further examples of this kind the simplicity and elegance of the complex treatment.

2. *Similar Triangles.* As can easily be verified, a necessary and sufficient condition for the equi-sensed similitude of two triangles $z_1z_2z_3, z_1'z_2'z_3'$ is the vanishing of the determinant

$$(1) \quad \begin{vmatrix} z_1 & z_1' & 1 \\ z_2 & z_2' & 1 \\ z_3 & z_3' & 1 \end{vmatrix} \equiv 0.$$

* Presented to the Society, Feb. 25, 1922.

† See Emch, *On a certain generation of rational circular and isotropic curves*, this BULLETIN, vol. 25, pp. 397-404 (1919), and also *On plane algebraic curves with a given system of foci*, same volume, pp. 157-161.

‡ *Leçons de Cinématique: Les imaginaires dans la cinématique du plan*, pp. 324-332 (1897).

§ *Vorlesungen über ausgewählte Gegenstände der Geometrie. Erstes Heft: Ebene analytische Kurven und zu ihnen gehörige Abbildungen*, pp. 1-18 (1911).