THE SIMPLE GROUP OF ORDER 2520

BY G. A. MILLER

Extract from a letter to F. N. Cole

If a second simple group of order $7!/2$ existed it could not be represented as a primitive group whose degree is less than twenty-one since all these primitive groups have been determined. The number of its subgroups of order 7 would be 120, for the only other divisor of $7!/2$ which is of the form $1 + 7k$ and greater than 20 is 36. It is easy to prove that such a simple group could not involve exactly 36 subgroups of order 7, as follows.

If a simple group of order $7!/2$ contained exactly 36 subgroups of order 7, it could be represented as a transitive group G on 36 letters representing the permutations of these 36 subgroups. Its subgroup G_1 composed of all its substitutions omitting one letter would be of order 70. It would therefore involve a cyclic subgroup of order 35 which would be regular, since the substitutions of order 7 would be regular. The subgroup G_1 could not be dihedral, since it could not involve negative substitutions. For the same reason the substitutions of order 2 could not transform the substitutions of order 5 into themselves and the substitutions of order 7 into their inverses. If these substitutions of order 2 could transform the substitutions of order 7 into themselves and the substitutions of order 5 into their inverses, G_1 would involve 5 conjugates of order 2. But this is impossible, since $36 \cdot 5$ is not divisible by 8.

Having proved that if the group in question existed it would contain 120 subgroups of order 7, we proceed to consider its subgroups of order 9. The number of these subgroups would be divisible by 35. In fact, if an operator of order 5 or an operator of order 7 could transform a subgroup of order 9 into itself it would be commutative with each of its operators. It was proved above that an operator of order 7 cannot be