Not all equations of that form are reducible to the form (10); for example

$$\Delta f'(x) + 2f'(x) + c(x)f(x) = 0$$

is not. However, equations with equal invariants which are not self-adjoint do not seem to be of great interest.

From the preceding theorem it follows that *if a self-adjoint equation of form* (10) *is of finite rank with respect to one of the transformations* (*S*) *or* (*T*), *it is of the same rank with respect to the other.*

Using the formula*

$$I_{S_n}(x) = I(x) + \sum_1^{n-1} [I(x+k) - J(x+k)] \\ - \Delta \frac{d}{dx} \log \left[\prod_{k=0}^{n-1} I(x+k) \prod_{k=0}^{n-2} I_{S_1}(x+k) \cdots I_{S_{n-1}}(x) \right]$$

and noting that for the self-adjoint case

$$I(x) = J(x) = -c(x),$$

we have

$$-c(x) = \Delta \frac{d}{dx} \log \left[\prod_{k=0}^{n-1} (-c(x+k)) \prod_{k=0}^{n-2} I_{S_1}(x+k) \cdots I_{S_{n-1}}(x) \right]$$

as a necessary and sufficient condition that the self-adjoint equation (10) *be of rank* $n+1$ *with respect to each of the transformations* (*S*) *and* (*T*).

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THE SECOND VOLUME OF VEBLEN AND YOUNG'S PROJECTIVE GEOMETRY.

Projective Geometry. By OSWALD VEBLEN and J. W. YOUNG. Boston, Ginn and Company; Vol. 2, by Oswald Veblen, 1918. 12 + 511 pages.

IN volume I, Veblen and Young were concerned particularly with those theorems of projective geometry which can be proved on the basis of their assumptions A of alignment, assumptions E of extension, and an assumption P of pro-

* Formula (8) in my thesis, l.c.