INTEGRO-DIFFERENTIAL EQUATIONS WITH CONSTANT LIMITS OF INTEGRATION.

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CONSIDER the linear integro-differential equation

$$\frac{\partial}{\partial \tau} u(\xi, \tau) = \int_a^b K(\xi, \eta) u(\eta, \tau) d\eta, \tag{1}$$

where τ is a real variable ranging over $|\tau - \tau_0| \leqq c$, and $K(\xi, \eta)$ is a continuous function defined in the square $a \leqq \xi \leqq b$, $a \leqq \eta \leqq b$. Volterra has shown* that the most general solution of this equation, reducing for $\tau = \tau_0$ to the arbitrary continuous function $u_0(\xi)$, is given by

$$u(\xi, \tau) = u_0(\xi) + \int_a^b L(\xi, \eta, \tau) u_0(\eta) d\eta, \tag{2}$$

where

$$L(\xi, \eta, \tau) = \sum_{n=1}^{\infty} \frac{(\tau - \tau_0)^n}{n!} K_n(\xi, \eta),$$

the functions K_n denoting, as usually, the iterated kernels of K with the understanding that $K_1 = K$.

It is proposed in this paper to give a solution of equation (1) in terms of the characteristic numbers and functions of the kernel K for the cases in which K is symmetric and skew-symmetric. Furthermore, some extensions of the theory to more general kernels will be pointed out and a formal analogy between integro-differential equations and partial differential equations will be shown. It is of interest to note that equation (1) may be considered as the limiting case of the finite system of differential equations with constant coefficients

$$\frac{du_i}{d\tau} = \sum_{j=1}^{n} K_{ij} u_j \qquad (i = 1, \cdots, n),$$

and the results for the integro-differential equations could be predicted from the well-known theory of this last system.†

* Volterra, *Lincei Rendiconti*, serie 5 (1914), 2d semester, p. 394.

† Schlesinger, "Zur Theorie der linearen Integrodifferentialgleichungen," *Jahresbericht der deutschen Mathematiker-Vereinigung*, vol. 24 (1915–16), p. 84.