The identity just preceding relation (5) is a generalization of that involved in the classic transformation of Abel; for, if we replace $v_1(x)$ by $g(x)$, $v_2(x)$ by $v(x)$, and $f(x)$ by 1, we have the Abel identity

$$\sum_{i=1}^{n} g(x_i)[v(x_i) - v(x_{i-1})]$$
$$\equiv - \sum_{i=1}^{n} v(x_{i-1})[g(x_i) - g(x_{i-1})] + g(b)v(b) - g(a)v(a).$$

It is obvious that a repeated use of (5) reduces the integral of $f(x)$ as to a product $v_1(x)v_2(x) \cdots v_n(x)$ to a sum of n integrals of functions as to $v_1(x)$, $v_2(x)$, $\cdots$, $v_n(x)$, respectively, under appropriate conditions like (6) and (7) and the hypothesis of the existence of these n integrals. The question arises naturally as to whether some simple identity exists, analogous to that employed in deriving (5), which would yield the entire result at once. It was through this question that I was led to identity (1). Having it, it is natural to extend the classic theorems about convergence of series previously obtained through the particular case which yields Abel's transformation. The reader will have no difficulty in obtaining through identity (1) the transformation of a Stieltjes integral mentioned at the beginning of this paragraph.

UNIVERSITY OF ILLINOIS,
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NOTE ON A PHYSICAL INTERPRETATION OF STIELTJES INTEGRALS.

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STIELTJES was led to his definition of integral by what he called the problem of moments (see §24 of his memoir in *Annales de la Faculté des Sciences de Toulouse*, 1894). Consider on a straight line OX a distribution of (positive) mass, the mass m_i being concentrated at the distance ξ_i from the origin O. The sum $\Sigma m_i \xi_i^k$ he called the moment of order k of the mass with respect to the origin. He also considered the more general distribution of mass on OX which is such that