THEOREM. *A necessary and sufficient condition that a non-singular matrix, M, shall be expressible as the product of two skew-symmetric matrices, viz., $M = S_1S_2$, is that every even invariant factor of the linear λ-matrix, $M - \lambda I$, shall be equal to the preceding odd invariant factor.**

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ON THE FIRST FACTOR OF THE CLASS NUMBER OF A CYCLOTOMIC FIELD.

BY MR. H. S. VANDIVER.

(Read before the American Mathematical Society April 27, 1918.)

LET l be an odd prime rational integer and consider the cyclotomic field defined by $e^{2i\pi/l}$. A number of questions connected with this field depend on the divisibility of its class number by l and its powers. This class number can be expressed as the product of two integral factors one of which (generally referred to as the first factor) is

$$h = \frac{f(Z)f(Z^3)\cdots f(Z^{l-2})}{(2l)^{\frac{1}{2}(l-3)}}, \tag{1}$$

where

$$f(x) = r_0 + r_1x + r_2x^2 + \cdots + r_{l-2}x^{l-2},$$

$Z = e^{2i\pi/l-1}$, r is a primitive root of l, and r_i is the least positive residue of r^i, modulo l.

Kummer† proved that the necessary and sufficient condition that h be divisible by l is that one of the numbers of Bernoulli, B_s, $[s = 1, 2, \cdots, (l-3)/2]$ is divisible by l, a B being termed divisible by an integer i when its denominator is prime to i and its numerator is divisible by i. Kronecker‡ gave another proof which was reproduced by Hilbert.§

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* Otherwise expressed, the condition is that the number of integers within parentheses in the characteristic shall always be even, and these alike in pairs. Thus [(2, 2); (3, 3, 1, 1); (1, 1)] is possible, while [(2, 1); (2, 1)]; [2], and [1, 1] are impossible.

† *Journal für die Mathematik*, vol. 40 (1850.)

‡ *Werke*, vol. 1, p. 93.

§ Die Theorie der algebraischen Zahlkörper, Bericht, p. 429.