Hence, quadratic forms exist for which the corresponding values of ρ satisfy these inequalities. However, no such form has ever been constructed for large values of n.

The problem of the closest *regular* (latticed) packing of spheres in R_n is equivalent to the problem of finding that positive definite quadratic form in n variables and of given determinant, say $D = 1$, whose least value, other than zero, is the highest possible for the given n and D (M$_4$, page 74 ff.). The ratio of the space occupied by spheres packed in regular layers in a large cube, say, to the volume of the whole cube, is indeed the number ρ defined above.

The author has recently proved, by an essentially different method, that no matter how the spheres be packed in a large volume V, in a "regular" fashion or not, the ratio of the space occupied by the spheres to the whole volume V is

$$< \frac{n/2 + 1}{2^{n/2}}.$$

It may be of interest to note in passing that it follows from the inequalities (1) satisfied by ρ for certain (though unknown) quadratic forms, that the shot-pile packing of spheres, though the closest packing in space of two dimensions and presumably also in space of three dimensions, is very far from being the closest packing in space of a large number of dimensions (M$_4$, page 95).

APPLICATIONS OF THE GEOMETRY OF NUMBERS TO ALGEBRAIC NUMBERS.

BY PROFESSOR L. E. DICKSON.

(Read at the Chicago Symposium of the American Mathematical Society, March 28, 1919.)

1. THE geometry of numbers not only furnishes a concrete geometric image of certain fundamental theorems on algebraic numbers, but also provides a new and attractive method of proving important theorems on algebraic fields. For the sake of concreteness we shall restrict attention to the typical case of the cubic field $F(\theta)$, which is composed of the numbers