# REPORT ON THE THEORY OF THE GEOMETRY OF NUMBERS.*

BY PROFESSOR H. F. BLICHFELDT.

(Read at the Chicago Symposium of the American Mathematical Society, March 28, 1919.)

1. *Introduction.*—There arises in theory of numbers an important class of problems of a kind best illustrated by an example: Consider the quadratic form $f = ax^2 + 2bxy + cy^2$, $ac - b^2 > 0$. Let the numerical value of the determinant $D = ac - b^2$ be given, but not the coefficients $a$, $b$, $c$ individually; also let it be specified that the variables $x$, $y$ must be integers (positive or negative or zero). Under these conditions, what can be predicted as to the least absolute value of $f$, other than $f = 0$? Designating this value by $[f]$, we have† $[f] \leqq 2/\sqrt{3}D^{\frac{1}{2}}$, the extreme limit being reached by the form $x^2 + xy + y^2$.

Isolated problems of like nature were studied by prominent mathematicians during the past century. Hermite discovered a superior limit to the least value $[f]$ of a positive definite quadratic form in $n$ variables in terms of $n$ and the numerical value of the determinant $D$: $[f] \leqq (\frac{4}{3})^{(n-1)/2}D^{1/n}$ (*Journal für Mathematik*, volume 40, 1850, page 263); this was the first important result of a general nature.

In the matter of references we shall use the abbreviations:

$M_1$, $M_2$, $M_3$, $M_4$ designate respectively the following books by Minkowski: "Diophantische Approximationen," Leipzig, 1907; "Geometrie der Zahlen," Leipzig, 1896–1910; "Gesammelte Abhandlungen," volumes 1 and 2, Leipzig, 1911.

$B_1$ refers to "A new principle in the geometry of numbers" by the author, *Transactions American Mathematical Society*, 1914, pages 227–235; $B_2$ to a paper read before this Society, San Francisco Section, April 6, 1918 (see this BULLETIN, 1918, page 418).

---

* An exposition of the theory of the geometry of numbers is to appear in the *Annals of Mathematics* during the fall and winter of the present year. For this reason the report here given of the lecture at the Chicago Symposium is very brief. Proofs have been omitted, and only a few illustrative examples are included. The fundamental theorems are, however, stated practically in full (§3).

† See $B_1$, p. 233, for references.