ON A THEOREM OF OSCILLATION.

BY PROFESSOR WILLIAM F. OSGOOD.

IN his book on series which represent the potential function Bôcher makes use of the first part of the following theorem.* All the variables are real.

THEOREM—*Let $\varphi(t)$ be continuous and monotonic in the interval $T \leqq t < \infty$, and let $\varphi(t)$ be always greater numerically than a certain positive constant γ:*

$$|\varphi(t)| > \gamma.$$

If $\varphi(t) < 0$, an arbitrary solution of the differential equation

$$\frac{d^2y}{dt^2} = \varphi(t)\, y,$$

oscillates an infinite number of times in any interval $T \leqq T' \leqq t < \infty$, in which it is considered, the amplitudes of the oscillations remaining finite.

If, furthermore, $\varphi(t)$ remains finite, the amplitudes of the oscillations do not become less than a certain positive constant. Moreover, the amplitudes vary monotonically, increasing when $\varphi(t)$ increases algebraically, and decreasing in the opposite case.

Bôcher states the theorem without the restriction that $\varphi(t)$ be monotonic, and outlines a suggestive dynamical proof. Professor Birkhoff's comment in the foregoing article led me to examine critically both theorem and proof. It is easy to see† that the theorem is not true under Bôcher's hypotheses. He needed the theorem, however, only in the restricted form above given. Moreover, he does not state the last paragraph of the theorem, this extension not being requisite for his purposes.

As regards this extension, it is not difficult to show that, if $\varphi(t)$ is not required to remain finite, the amplitudes of the oscillations may become infinitesimal, when t becomes infinite.

---

* Ueber die Reihenentwickelungen der Potentialtheorie; Teubner, 1894, p. 178.

† Cf. infra.