index 8 whenever $n > 4$, and that the corresponding quotient group is the octic group. Hence K involves exactly three subgroups of index 2 whenever n exceeds 4 and only two other invariants subgroups besides identity, viz., the mentioned subgroup of index 8 and one of index 4 corresponding to the invariant subgroup of order 2 of the octic group. These results apply also to the special case when $n = 3$.

TRANSLATION SURFACES IN HYPERSPACE.

BY PROFESSOR C. L. E. MOORE.

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1. If the rectangular coordinates of the points of a surface can be expressed in the parametric form

$$(1) \qquad x_i = f_i(u) + g_i(v) \qquad (i = 1, 2, \ldots, n),$$

where f_i are functions of u alone and g_i functions of v alone, the surface is called a translation surface. It is seen that a translation can be found which will send any parameter curve $u =$ const. into any other one of the same system. The same is true of the curves $v =$ const. The surface (1) is also seen to be the locus of the mid-points of the lines joining the points of

$$(2) \quad C_1:\ x_i = 2g_i(u) \quad \text{to the points of} \quad C_2:\ x_i = 2f_i(v).$$

The character of the surface can then be determined, in a great measure, by the form and relative position of these two curves. Nearly all writers on surface theory* mention three facts concerning translation surfaces in 3-space:

(a) The generators of the developable which touches the surface along a curve $u =$ const. are tangent to the curves $v =$ const., or in other words the directions of the parameter curves passing through a given point are conjugate directions.

(b) There are surfaces which can be expressed in more than one way in the form (1).

* Darboux, Théorie générale des Surfaces, vol. 1, pp. 148, 340. Scheffers, Theorie der Flächen, vol. 2, pp. 188, 245.