# DETERMINANT GROUPS.

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## §1. *Introduction.*

LET $D$ represent a determinant of order $n$ whose $n^2$ elements are regarded as independent variables. The substitutions on these $n^2$ elements which transform $D$ into itself constitute a substitution group $G$, which we shall call the *determinant group* of degree $n^2$. As the elements of $D$ are supposed to be independent variables, it results from the definition of a determinant that every substitution of $G$ must transform the elements of $D$ in such a manner that all the elements of a line (row or column) appear in a line after the transformation.

Hence the substitutions of $G$ correspond to the permutations of the elements of $D$ resulting from transforming its rows and columns independently according to the alternating group of degree $n$, transforming its rows and columns simultaneously according to negative substitutions in the symmetric group of this degree, and interchanging the rows and columns. The order of $G$ is therefore $(n!)^2$, and hence the number of the distinct determinants that can be formed by permuting the $n^2$ elements of $D$ is $n^2!/(n!)^2$.* These determinants may be arranged in pairs such that each pair is composed of the determinants which differ only with respect to sign. In particular, the square of $D$ is transformed into itself by a group $K$ whose order is twice the order of $G$ and which contains $G$ as an invariant subgroup.

Some of the abstract properties of $G$ follow directly from the fact that it is simply isomorphic with the imprimitive substitution group of degree $2n$ whose head is composed of the positive substitutions in the direct product of two symmetric groups of degree $n$. These substitutions correspond to interchanges of the rows among themselves and the columns among themselves or a combination of such interchanges. The re-

* G. Bagnera, *Giornale di Matematiche*, vol. 25 (1887), p. 228; it may be noted that in the review of this article in *Jahrbuch über die Fortschritte der Mathematik* the author's name appears in the form Bergnera.