INVOLUTIONS ON THE RATIONAL CUBIC.

BY PROFESSOR R. M. WINGER.

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Introduction.

1. THE general subject of involution as applied to rational curves has been widely studied, notably by Weyr, Stahl, Coble and many Italian writers. It is the purpose of this paper to discuss certain involutions on the rational cubic, R^3.

If s_i denote the elementary symmetric functions of coordinates $x_1, x_2, \ldots, x_n$ of n points (elements in the binary domain), the most general involution of order n, $I_{n-1.1}$, i. e., one in which $n - 1$ points of a set determine the remaining one, will be defined by

$$(1) \qquad a_0 s_n + a_1 s_{n-1} + a_2 s_{n-2} + \cdots + a_{n-1} s_1 + a_n = 0.$$

The involution is thus made up of all sets of n points apolar to a fixed set, the n-fold points of the involution, given by

$$(2) \qquad a_0 x^n + \binom{n}{1} a_1 x^{n-1} + \binom{n}{2} a_2 x^{n-2} + \cdots + \binom{n}{n-1} a_{n-1} x + a_n = 0.$$

The following alternative and equivalent definition is serviceable when the n points of a set are represented implicitly by an equation: An $I_{n-1,1}$ is an $(n-1)$-parameter family of binary forms of order n

$$(3) \qquad f_0 + k_1 f_1 + k_2 f_2 + \cdots + k_{n-1} f_{n-1}.$$

More generally, if $n - r$ points of a set suffice to determine the remaining r, x_i must satisfy r equations of the type (1) and (3) reduces to an $(n - r)$-parameter family. The corresponding involution is denoted by $I_{n-r,\,r}$.

2. Choosing for triangle of reference the nodal tangents and the line of flexes, the equation of the curve may be written in the canonical form

$$(4) \qquad x_1 = 3t^2, \qquad x_2 = 3t, \qquad x_3 = t^3 + 1.$$