where subscripts indicate partial differentiation, and where e is chosen equal to ± 1 so as to make A positive. Differentiating A with respect to θ, and setting the result equal to zero, we get

$$(1) \quad F(-\phi_x \sin\theta + \phi_y \cos\theta) - (\phi_x \cos\theta + \phi_y \sin\theta)(-F_{x'} \sin\theta + F_{y'} \cos\theta) = 0,$$

$F_{x'}$, $F_{y'}$ denoting partial derivatives of F with respect to its third and fourth arguments respectively. Since

$$F = F_{x'} \cos\theta + F_{y'} \sin\theta,*$$

equation (1) reduces to

$$(2) \quad \phi_y(x, y)F_{x'}(x, y, \cos\theta, \sin\theta) - \phi_x(x, y)F_{y'}(x, y, \cos\theta, \sin\theta) = 0,$$

and if we define direction on the curve $\phi = c$ by means of the angle $\bar{\theta} = \arctan(-\phi_x/\phi_y)$, (2) becomes

$$F_{x'}(x, y, \cos\theta, \sin\theta) \cos\bar{\theta} - F_{y'}(x, y, \cos\theta, \sin\theta) \sin\bar{\theta} = 0.$$

But this equation determines the value of θ to which the curve $\phi = c$ is transversal.†

Therefore the differential quotient $d\phi/dS$ is equal to zero in the direction tangent to the curve $\phi = c$ and has its maximum absolute value in the direction to which the curve $\phi = c$ is transversal.

WASHINGTON UNIVERSITY,
ST. LOUIS, MO.

TANGENTIAL INTERPOLATION OF ORDINATES AMONG AREAS.

BY DR. C. H. FORSYTH.

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IF we wish to interpolate several values in each interval between the successive ordinates $u_0, u_1, u_2, \cdots, u_n$ by finite differences, only a low order of differences can with propriety be used, since high orders based on ordinary statistical data

* See Bolza, loc. cit., p. 196.
† See Bolza, loc. cit., p. 303.