ADDITIVE FUNCTIONS OF A POINT SET.

Intégrales de Lebesgue, Fonctions d'Ensemble, Classes de Baire. Par C. DE LA VALLÉE POUSSIN. Paris, Gauthier-Villars, 1916. viii + 154 pp.

THE general notion of an additive function of a point set is one of the most important introduced by the present French school of mathematicians. It is due to H. Lebesgue. An additive function of a point set is one whose value on a sum of sets is the sum of its values on each term. These terms, the number of which may be infinite, have in pairs no common element. The volume under review (one of the Borel monographs) is devoted to the theory of additive functions of a point set; it contains the matter of lectures delivered at the College of France between December, 1915, and March, 1916. The author had already treated the problem in his Harvard lectures, which were partially published in 1915 in the *Transactions of the American Mathematical Society.* Some of the results had also appeared in volume II of his Cours d'Analyse, third edition. In the present volume the treatment is rendered distinctly more complete and satisfying through the use of new methods and the derivation of new results.

In these lectures we have a careful analysis of the notion of additivity and the derivation of the consequences, singularly precise and interesting, which follow from the sole property of a function implied in this notion.

The simplest and earliest known additive function of a point set is its measure. The definition was given by Borel in 1898. It furnishes the point of departure for the entire theory of additive functions. It is this function which is considered in the beginning of the first of the three (nearly equal) parts of the present monograph. (Lebesgue integrals are treated in the latter portion of this part.) The measure is a non-negative function whose value is given on certain point sets called elementary figures and is then to be defined on other sets in such way as to satisfy the requirement of being additive. The point sets on which this requirement is satisfied are then the measurable sets.

If a function possesses the property of being additive only