AN UNDERVALUED KIRKMAN PAPER.

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THE purpose of this note is to emphasize the importance in the theory of triad systems of a Kirkman* paper, which appears to have been overlooked by all writers on this subject up to the present time.

A short explanation of the symbols employed in the paper is necessary. The symbol $Q_{x,y,z}$ denotes the greatest number of combinations that can be made with x elements, y at a time, so that no combination of z elements shall be twice employed; for brevity $Q_{x,3,2}$ is replaced by Q_x. The symbol V_x denotes the number of pairs possible with x elements that are excluded from Q_x. The symbol q_x denotes the number of triads formed with x elements, in which no duad is twice employed, q_x being not necessarily a maximum; v_x is the number of duads possible with x elements not employed in q_x. Four pairs such as 12, 23, 34, 41 forming a closed circle are denoted by the symbol C_4.

The object of Kirkman's paper is to determine the value of Q_x, and to establish the following theorems:

$$Q_x = \frac{x(x-1)}{6} - \frac{V_x}{3},$$

where

$$V_x = \frac{x}{2} + 3k + 1 \text{ if } x = 6n - 2;$$

$$V_x = 6k + 4 \quad \text{“ } x = 6n - 1;$$

$$V_x = \frac{x}{2} \quad \text{“ } x = 6n,\ 6n + 2;$$

$$V_x = 0 \quad \text{“ } x = 6n + 1,\ 6n + 3;$$

$2^m(2k+1) = n$; n, m, x, k are integers $\geqq 0$.

The case of special interest to us is that in which $V_x = 0$,

* *Cambridge and Dublin Math. Journal*, vol. 2 (1847).