The system of groups under consideration is of special interest because it includes $p - 2$ non-abelian groups in which every operator except identity is of order p, for every possible value of the prime number p. The number of these groups for a particular prime number therefore depends upon this prime. The fact that the group generated by s_1, s_2 is completely determined by its order seems also worth noting.

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A THEOREM IN THE ANALYSIS OF REAL VARIABLES.

BY PROFESSOR R. L. BORGER.

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IN this paper the following theorem is proved:

THEOREM: *If the two real functions $U(x, y)$, $V(x, y)$ of the real variables x, y satisfy the following conditions at each point of a closed region R:*

(*a*) *U and V continuous in x and y jointly;*

$$(b) \qquad \frac{\partial U}{\partial x} \equiv U_1, \quad \frac{\partial U}{\partial y} \equiv U_2, \quad \frac{\partial V}{\partial x} \equiv V_1, \quad \frac{\partial V}{\partial y} \equiv V_2$$

exist and are finite;

(*c*) $\Delta U = hU_1 + kU_2 + \rho_1(h, k), \quad \Delta V = hV_1 + kV_2 + \rho_2(h, k)$;

(*d*) $\displaystyle \lim_{h,\, k \doteq 0} \frac{|\rho_i(h, k)|}{|h| + |k|} = 0 \quad (i = 1, 2)$;

(*e*) $U_1 = V_2, \quad U_2 = - V_1$;

then U and V are analytic functions of x, y in R.

An immediate consequence of the theorem is that if any function W of a complex variable z possesses a finite derivative at each point of a simply connected closed region R then:
1. This derivative is continuous.
2. All the derivatives of W exist.
3. The function W may be represented by a power series in z.