

on any set E and

$$\int_E dx = \overline{\text{meas. } E}.$$

It is questionable *whether this precise formula is a decided improvement over M. Lebesgue's statement.* But, further, from this formula it is deduced that *the Pierpont integral does not enjoy the fundamental property that if E, F are sets with no points in common*

$$\int_{E+F} f(x)dx = \int_E f(x)dx + \int_F f(x)dx$$

(which however is true when E, F are "separated," according to Professor Pierpont). It suffices to apply this formula when $f(x) \equiv 1$, $E + F$ is an interval and E is non-measurable.

A REPLY TO A REPLY.

BY PROFESSOR JAMES PIERPONT.

As I view the issue between Professor Fréchet and myself, it may be summed up as follows:

1°. Professor Fréchet thought that it was possible to split a measurable set into two *separated* non-measurable sets, and he gave an alleged example. Since no such division is possible this example proved to be an ignis fatuus.

2°. Supported by this example, it was easy for Professor Fréchet to bring a number of grave charges against my work, in fact it might seem as if my whole theory had toppled to the ground.

3°. Professor Fréchet now admits (provisionally) that he was in error on this score, but he still holds to his "original assertion" that my integral definition "is inappropriate," "though for partly different reasons." What are these new reasons? Although I have read and reread the above article I have found but one, viz.: Suppose A is *non-measurable* and suppose B and C form a *non-separated* division of A , then the relation

$$(1) \quad \int_A = \int_B + \int_C$$

may not hold.