RELATIONS AMONG PARAMETERS ALONG THE RATIONAL CUBIC CURVE.

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Introduction.

THE purpose of this paper is to give the proofs of two new theorems concerning relations among sets of parameters along the rational plane cubic curve. The first theorem concerns a projective relation possessed by the parameters of the four residual points in which the osculant conic at any point meets the cubic. The second theorem defines the relation which exists among the parameters of the four tangents drawn from any point of the plane to the rational cubic. The proof depends upon a method of deriving the parametric equations of the node of the rational cubic.

We shall call the rational plane cubic the R^3, and write its parametric equations in the form

$$(1) \qquad x_i = (a_it)^3 \equiv a_it^3 + 3b_it^2 + 3c_it + d_i \quad (i = 0, 1, 2).$$

§ 1. *The Osculant Conic and the Associated Theorem*

A point on the osculant conic of the R^3 at a point whose parameter is t' is defined by the equations

$$(2) \quad x_i = (\alpha_it')(\alpha_it)^2 \equiv (a_it' + b_i)t^2 + 2(b_it' + c_i)t + (c_it' + d_i) \quad (i = 0, 1, 2).$$

The equation* of the osculant conic at t has the form

$$(3) \quad [4|abx||bcx| - |acx|^2]t^4 + [4|abx||bdx| - 2|acx||adx| + 2|acx||bcx|]t^3 + (\cdots)t^2 + (\cdots)t + (\cdots) = 0.$$

In particular, if $t = 0$, (3) becomes

$$(4) \qquad 4|bcx||cdx| - |bdx|^2 = 0.$$

If the values of x_i from (1) are substituted in (4), and the factor t^2 is removed, the result is

$$(5) \quad [4|abc|\,|acd| - |abd|^2]t^4 + 12|abc|\,|bcd|t^3 + 6|abd|\,|bcd|t^2 + 4|acd|\,|bcd|t + 3|bcd|^2 = 0.$$

* J. E. Rowe, *Messenger of Math.*, No. 512 (Dec., 1913), pp. 118–119.