LIMITS OF THE DEGREE OF TRANSITIVITY OF SUBSTITUTION GROUPS.

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THE main object of the present paper is to establish an elementary theorem which gives always a smaller upper limit for the degree of transitivity of a substitution group of degree $n > 12$ which does not include the alternating group of this degree, than the one given by the commonly quoted theorem that this limit cannot exceed $\frac{1}{3}n + 1$.* The theorem to be established is a generalization of the one published by the present writer in volume 4, page 140, of this BULLETIN. In Pascal's Repertorium, loc. cit., a footnote states that the limit $\frac{1}{3}n + 1$ is actually attained by the five-fold transitive Mathieu group of degree 12. In view of the results of the present paper this footnote could be completed by adding that this limit cannot be attained for any degree which exceeds 12. It is clearly also attained when $n = 6$, although this is not mentioned in the footnote.

Let G be any transitive substitution group of degree $n = kp + r$, where p is a prime number such that $p > k$, and $r > k$, and all the symbols p, r, k represent positive integers. In what follows it will always be assumed that G is neither alternating nor symmetric on the n letters and that $k > 1$. If G is more than r-fold transitive it includes a transitive subgroup H of degree kp, and hence its order is divisible by p. A Sylow subgroup of order p^α contained in H must be intransitive and each of its transitive constituents must be of degree p, since G cannot involve a substitution composed of a single cycle of degree p, according to the well-known theorem that a primitive group which involves a cyclic substitution of degree p cannot be of degree greater than $p + 2$ unless it includes the alternating group of its own degree.

It may be assumed that H is composed of all the substitutions of G on a certain set of kp letters. Since it is assumed that

* Cf. Pascal's Repertorium der höheren Mathematik, vol. 1 (1910), p. 211; Encyclopédie des Sciences mathématiques, tome 1, vol. 1, p. 549; etc.