enter linearly and homogeneously, found their finite equations, and introduced variables such that each group becomes its own parameter group. The resulting groups (l. c., page 648, bottom, and page 649) are our h and g. From these he derived the above two algebras.

6. Scheffers' determination (pages 654–6) of the algebra of quaternions is based upon the existence of the group of transformations t'_Y of § 4. In a rather arbitrary manner he selected four infinitesimal transformations out of an aggregate of the ∞^6 infinitesimal automorphs of the quadric surface, and verified that the four generate a four-parameter group. The guide to this seemingly fortunate selection may well have been the previous knowledge of the group defined by the algebra of quaternions. The above discussion in § 4 not only gives a natural derivation of quaternions from the theory of groups but leads to the total group of automorphs of a quadric surface and not merely to its continuous subgroup.

THE UNIVERSITY OF CHICAGO.

AN ASPECT OF THE LINEAR CONGRUENCE WITH APPLICATIONS TO THE THEORY OF FERMAT'S QUOTIENT.

BY MR. H. S. VANDIVER.

(Read before the American Mathematical Society, August 4, 1915.)

IN 1903, Professor G. D. Birkhoff communicated to me the following theorem:

If p is a prime integer and a is a positive integer prime to p, then there is at least one and not more than two sets (x, y) such that

$$a \equiv \pm x/y \pmod p$$

where x and y are integers prime to each other and $0 < x < \sqrt{p}$, $0 < y < \sqrt{p}$.

Professor Birkhoff has kindly allowed me to use this result, and in the present paper I shall give a proof of the theorem which involves a continued fraction algorithm for a direct determination of each set. Some extensions and applications are also given.