ON THE RELATION BETWEEN LINEAR ALGEBRAS AND CONTINUOUS GROUPS.

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1. THE aim of this note is to give a very elementary account of the mutual relation between any linear associative algebra (system of hypercomplex numbers) and a type of continuous groups, without presupposing on the part of the reader a knowledge of either subject. The relation in question, first observed by Poincaré, enables us to translate the concepts and theorems of the one subject into the language of the other subject. It not only doubles our total knowledge, but gives us a better insight into either subject by exhibiting it from a new point of view. Incidentally, we shall obtain several other results of general interest.

2. To begin with the simplest illustration, we set up a correspondence between each real number c, not zero, and the transformation $z' = cz$, denoted by T_c, on the real variable z. The result of applying in succession T_c and the new transformation $T_{c'}$ (which we may express in the form $z'' = c'z'$) is the same as applying the single transformation $z'' = (c'c)z$. Hence we say that the *product* $T_cT_{c'}$ of the two given transformations is the transformation $T_{c''}$, where $c'' = c'c$. The set of transformations which correspond to the system (or algebra) of all real numbers, other than zero, is said to form a *group* G since the product of any two of these transformations is a transformation of the same set. In particular, G is a one-parameter continuous group. The relation $c'' = c'c$ between the parameters in $T_cT_{c'} = T_{c''}$ defines a transformation of c into c'' with the parameter c'. Since c' ranges over all real numbers other than zero, the resulting transformations $c'' = c'c$ on the parameters form a group which is the same as G, apart from the notation of the variables. Hence G is said to be its own parameter group.

Next, let z denote a complex variable $x + yi$ and let c range over all complex numbers $a + bi$ other than zero. Then T_c is equivalent to the binary transformation

$$T_{a,\,b}: \qquad x' = ax - by, \quad y' = bx + ay.$$