$$(g - av)_p = \lim_{S \doteq p} \frac{1}{\sigma} \int_S \left[\frac{\partial}{\partial x} (a_{11}v) + \frac{\partial}{\partial y} (a_{12}v) - a_1 v \right] dy - \left[\frac{\partial}{\partial x} (a_{21}v) + \frac{\partial}{\partial y} (a_{22}v) - a_2 v \right] dx,$$

then

$$(vf - ug)_p = \lim_{S \doteq p} \frac{1}{\sigma} \int_S \left[a_{11} \left(v \frac{\partial u}{\partial x} - u \frac{\partial v}{\partial x} \right) + a_{12} \left(v \frac{\partial u}{\partial y} - u \frac{\partial v}{\partial y} \right) + \frac{a_1 - b_1}{2} uv \right] dy - \left[a_{21} \left(v \frac{\partial u}{\partial x} - u \frac{\partial v}{\partial x} \right) + a_{22} \left(v \frac{\partial u}{\partial y} - u \frac{\partial v}{\partial y} \right) + \frac{a_2 - b_2}{2} uv \right] dx,$$

where p is any point within S.

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CONVERGENCE OF THE SERIES $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{x^i y^j}{i - j\gamma}$ (γ IRRATIONAL).

BY PROFESSOR W. D. MACMILLAN.

(Read before the American Mathematical Society April 2, 1915.)

The method of proof which is here used depends upon the properties of continued fractions. Any irrational number γ can be expanded as a simple continued fraction

$$\gamma = a_1 + \frac{1}{a_2 +} \frac{1}{a_3 +} \frac{1}{a_4 +} \cdots.$$

Let p_n/q_n be the nth principal convergent,* and P/Q be any intermediate convergent lying between p_{n-2}/q_{n-2} and p_n/q_n. Then

$$\frac{p_{n-2}}{q_{n-2}} < \frac{P}{Q} < \frac{p_n}{q_n} < \gamma < \frac{p_{n+1}}{q_{n+1}} < \frac{p_{n-1}}{q_{n-1}}$$

if n is odd, and

* The notation used here agrees with that of Chrystal's Algebra, Vol. II, Chap. XXXII.