

THE RÔLE OF THE POINT-SET THEORY IN
GEOMETRY AND DYNAMICS.

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IN his "Child's Garden of Verses" Robert Louis Stevenson says simply but poetically:

"The world is so full of a number of things,
I'm sure we should all be as happy as kings."

This catches the spirit of the Mengenlehre, and may well be taken as its motto. In more homely phrase the mathematician and physicist declare that the world is made up of an *infinite* number of things. Hence in thought and nature we have to consider numbers which are infinite. Even though they be not actually infinite, they are so enormously, unthinkably large that they can be most advantageously handled by regarding them as infinite. Such a mode of treatment was, of course, foreign to Greek thought. But, as you all know, the classic Greek mathematics was straitened by its failure to admit the infinitesimally small and the infinitely great magnitudes of the calculus. In somewhat similar fashion modern mathematics was long hampered by the lack of a mathematical theory, not of the potentially infinite quantity of the calculus, but of actually infinite aggregates. It was the incalculable and distinctive service of Georg Cantor to have perceived the urgent need of such a theory. Minerva-like and full panoplied it sprang from his teeming brain. Already its achievements have been very great, but it is far from maturity and its full powers are still to be revealed.

The influence of this new theory of infinite aggregates was first decisively felt in the theory of functions of the real and complex variables, where infinite sets of points, often irregular in character, frequently present themselves; for example, the discontinuities of an integrable function or the singularities of an analytic function, either of which may occur in infinite number. Here also, in the function-theory, its richest fruition