$$N - 4p - (m - 2) + 2r - \rho = \bar{I} + 2r - \bar{\rho} + 2,$$

where $\bar{\rho}$, $\bar{I}$, $\bar{\rho}$ are the invariants $\rho$ and those of Zeuthen-Segre for $\bar{F}$. If $I$ is the same invariant for $F$, since $\bar{I} - \rho = I - \rho$, we have

$$I = N - 4p - m = \Sigma s - 4p - m.$$

Hence $\Sigma s$ is equal to the "equivalence" in nodes of the point $(a, b, c)$ in the evaluation of the invariant $I$ for $F$ by means of the pencil $C_y$. This property can be shown directly for the following cases: 1°. $F$ has only ordinary nodes. 2°. In the vicinity of any of these nodes there lie other nodes, or ordinary infinitesimal multiple curves. In these cases it is easy to show that in the vicinity of the nodes all the numbers such as $h$ are equal to 2. It would be interesting to know if such is always the case, but the preceding investigation shows that for the applications this does not matter.

UNIVERSITY OF KANSAS,
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## THE FOURTH DIMENSION.

*Geometry of Four Dimensions.* By HENRY PARKER MANNING, Ph.D. New York, The Macmillan Company, 1914. 8vo. 348 pp.

EVERY professional mathematician must hold himself at all times in readiness to answer certain standard questions of perennial interest to the layman. "What is, or what are quaternions?" "What are least squares?" but especially, "Well, have you discovered the fourth dimension yet?"

This last query is the most difficult of the three, suggesting forcibly the sophists' riddle "Have you ceased beating your mother?" The fact is that there is no common locus standi for questioner and questioned. To the professional mathematician the fourth dimension usually suggests a manifold of objects depending upon four independent parameters, which it is convenient to describe in geometrical language. Occasionally he does not make any use of analysis, but builds up what the Italians call a "Sistema ipotetico deduttivo" of abstract assumptions and conclusions. The whole thing is professional, and unromantic. Such ideas, are, naturally,