

A great many theorems dealing with the limit-behavior of functions were obtained by Du Bois-Reymond in a series of papers dating from 1871 to 1880. These theorems have been collected, recast according to modern requirements of rigor, and amplified by Hardy in No. 12 of the Cambridge Tracts in Mathematics and Mathematical Physics. The work has been done in a manner admitting of no criticism; the treatment is clear and readable; the proofs are accurate and carefully worded.

Questions of the sort considered are generally reducible to a comparison of the rates of increase of positive real functions, as the positive real independent variable becomes infinite (continuously or over integral values); into this form most of the results are thrown by the author. A considerable part of the tract deals with logarithmico-exponential functions,—those obtained by rational operations, the extraction of roots, and the taking of logarithms and exponentials. Such functions have various properties which render their study easy and important; for instance, they always admit of comparison as regards rate of increase, and relations of comparison may under simple conditions be differentiated and integrated.

The tract contains a sketch of applications to such questions as the convergence of series and integrals, asymptotic formulas, the distribution of prime numbers, and the theory of integral functions of a complex variable.

One is impelled to wonder how much of the fairly extensive notation introduced will be found really desirable in actual use of the results. The notions of inferior and superior limit have won a permanent place; Landau's symbols  $O(f)$  and  $o(f)$  have in a short time come into such wide use as apparently to insure their retention. It is doubtful whether any further notation will be found necessary.

The tract is printed with the clearness characteristic of the series. The only typographical inaccuracy noted is on page 42, line 12, where  $e_{\Delta x}$  should be replaced by  $e^{\Delta x}$ .

WALLIE ABRAHAM HURWITZ.

*The Hindu-Arabic Numerals.* By DAVID EUGENE SMITH and LOUIS CHARLES KARPINSKI. Boston, Ginn and Company, 1911. vi + 160 pp.

THE origin and development of our present number system is probably given as little thought as anything that is so