

If now  $p$  is Fréchet interior to  $\mathfrak{S}$  and is a limiting element of  $\mathfrak{I}$  a subclass of  $\mathfrak{R}$ , then, by definition of Fréchet interior, limiting element, and  $L^2$ ,  $\mathfrak{S}$  contains an infinity of elements of  $\mathfrak{I}$ . Therefore  $p$  is interior to  $\mathfrak{S}$  in the sense of § 1. Furthermore if  $p$  is interior ( $\mathfrak{R}$ ) to  $\mathfrak{S}$  in the sense of § 1, then  $\mathfrak{S}$  contains an element  $q$  (distinct from  $\mathfrak{B}$ ) of every subclass  $\mathfrak{I}$  of  $\mathfrak{R}$  for which  $p$  is a limiting element. Then if  $p = L_n r_n$  (distinct)  $p$  is a limiting element of the class  $[r_n]$ . Hence  $\mathfrak{S}$  contains  $r_{n_1}$  distinct from  $p$ . Since  $p$  is a limiting element of the class obtained from  $[r_n]$  by removing  $r_{n_1}$  ( $L^2$ ) it is evident that at most a finite number of elements of  $[r_n]$  are not in  $\mathfrak{S}$ . Therefore  $[r_n]$  is ultimately contained in  $\mathfrak{S}$ .

T. H. Hildebrandt\* has given a definition of interior ( $\mathfrak{R}$ ) which becomes equivalent to the Fréchet interior ( $\mathfrak{R}$ ) for systems ( $\mathfrak{B}; L^{123}$ ). This definition omits the condition that the sequence  $\{r_n\}$  consist of distinct elements. If then  $p = L_n r_n$  and  $r_{n_0}$  is repeated infinitely often, in a system ( $\mathfrak{B}; L^{123}$ ),  $r_{n_0} = p$ . That  $r_{n_0}$  is contained in any class  $\mathfrak{S}$  to which  $p$  is Fréchet interior ( $\mathfrak{R}$ ) is evident. A restatement of Theorem IV for systems ( $\mathfrak{B}; L^{123}$ ) gives us a generalization of a theorem of Hildebrandt.†

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## COMPLETE EXISTENTIAL THEORY OF SHEFFER'S POSTULATES FOR BOOLEAN ALGEBRAS.

BY PROFESSOR L. L. DINES.

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IN a recent number of the *Transactions* Sheffer‡ presented an elegant and concise set of five postulates for Boolean algebras, and proved them mutually consistent and independent. Professor E. H. Moore§ has suggested a further interesting problem in connection with such sets of postulates, namely the determination of all general implicational relations

\* Loc. cit., p. 268 (10).

† Loc. cit., p. 282 (2).

‡ H. M. Sheffer, "A set of five postulates for Boolean algebras with application to logical constants," *Transactions*, vol. 14 (1913), pp. 481-488.

§ E. H. Moore, "Introduction to a form of general analysis," *New Haven Mathematical Colloquium*, Yale University Press, page 82.