

SHORTER NOTICES.

Grundzüge der Differential- und Integralrechnung. Von Dr. GERHARD KOWALEWSKI, a. o. Professor der Mathematik an der Universität Bonn. Leipzig und Berlin, Teubner, 1909. 452 pp.

It is the purpose of this book to present a rigorous treatment of the foundations of the calculus. The author therefore begins with a consideration of numbers. Assuming a knowledge of the rational numbers, he defines the irrational number by means of the Dedekind cut, and in the first three chapters, develops the properties of the real number system. In Chapter II (page 14) he defines a limit of a sequence in this way: "Steht eine Folge u_1, u_2, u_3, \dots zu einer Zahl u in der Beziehung, dass in *jeder* Umgebung von u fast alle Glieder der Folge liegen, so nennt man die Folge konvergent und u ihren Grenzwert." In this definition, he uses the words "fast alle" in the sense of "with a finite number of exceptions." In his notation $\lim u_n = u$ means that there exist u 's of the set u_n in every neighborhood of u , with a finite number of exceptions. The equivalence of this definition with the usual form of ϵ statement is established later. Chapter IV deals with functions and variables. y is a function of x when to each x there corresponds a single y . The principal subjects contained in the chapter are the concepts of continuity, the definition and properties of the functions a^x , $\log x$, and finally the derivation of the $\lim_{n \rightarrow \infty} (1 + 1/n)^n$. The development of this limit is effected with unusual freedom from artifice by means of the inequality

$$(1 + h)^n > 1 + nh \quad (1 + h > 0).$$

The sine and cosine are defined by means of the infinite series, and in preparation for this introduction Chapter VII gives a rather full treatment of the elementary properties of infinite series, concluding with the proof of the theorem on the differentiation of a power series.

Chapters IX and XI are devoted to the theory of maxima and minima of functions of one and two variables, the criteria for a maximum or a minimum for a function of a