congruence $1^{p-2} + 2^{p-2} + \cdots + \left(\frac{p-1}{2}\right)^{p-2} \equiv 0 \pmod{p}$. Professor Putnam discusses the general congruence of this type, $1^r + 2^r + \cdots + \left(\frac{p-1}{2}\right)^r \equiv a \pmod{p}$, and shows that by expressing a as a fraction it may be given a value for any fixed r (less than p) that is independent of p.

5. In a paper in the *Proceedings of the Edinburgh Mathematical Society,* Professor Allardice considered a geometrical transformation in the plane, $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, where ϑ is the angle formed by the enveloping tangents of a curve with a given straight line l, the axis of transformation, and φ is the angle formed by l and t, a system of lines through the intersection of l and t, which envelop the transformed curve of c. In the present paper, Dr. Stager considers analytically a similar transformation in space and applies it to certain systems of spheres. The method of transformation applied to space is as follows: Let α be a given plane and P be any solid. Further, let a plane β be tangent to P and intersect α in i, making with α an angle ϑ. If through i we draw a plane β', making with α an angle φ, such that $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, the envelop of β' is defined as the "transform of P." The paper concludes with a number of applications of the method.

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THE TOTAL VARIATION IN THE ISOPERIMETRIC PROBLEM WITH VARIABLE END POINTS.

BY DR. A. R. CRATHORNE.

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In the simple problem of the calculus of variations,

$$J = \int_{x_1}^{x_2} F(x, y, x', y')dt = \text{minimum},$$

the total variation can be expressed as an integral of which the integrand is the Weierstrassian E-function. It is the object of this note to express in a similar way the total vari-