double root and consequently the line is tangent to the surface $f = 0$. Thus all the lines which belong to the double factor of Φ are tangent to the surface $f = 0$.

Now let the x_i and y_i have such values that they satisfy $\Phi = 0$ but the point x_i does not necessarily lie on $f = 0$, and consider the totality of lines $v_i = x_i + y_i\tau$ which are tangent to $f = 0$. They are given by the equations

$$(12) \qquad f(x_i + y_i\tau) = 0, \quad \frac{\partial}{\partial\tau} f(x_i + y_i\tau) = 0.$$

The τ-eliminant of equations (12) represents the totality of lines tangent to $f = 0$. Hence it includes the two or more factors of Φ which become equal when the x_i satisfy $f = 0$. Since the eliminant is rational and Φ is irreducible, the eliminant must be Φ itself or a multiple of it.

In the astronomical problem the equation $F = 0$ which defines the roots s_j is known. The surfaces $f = 0$ are therefore readily determined and all possible functions Φ can be found. To satisfy the conditions which Bruns has stated, Φ must be factorable into real factors which are polynomials in the y_i and rational in the x_i and s. It is found upon examination that there does not exist a Φ which satisfies all these conditions and consequently the original φ_0 with which we set out cannot contain s. Therefore Bruns' conclusion that we need consider only integrals of the type (2) was correct, even though his argument was wrong. The integrity of the theorem has been preserved by the penetrating insight of Poincaré.

UNIVERSITY OF CHICAGO,
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NOTE ON THE GROUPS FOR TRIPLE-SYSTEMS.

BY MISS L. D. CUMMINGS.

THE method of "Triple-systems as transformations and their paths among triads," given by Professor White in the *Transactions*, volume 14 (1913), page 6, has been applied by me to the two following triple-systems on fifteen elements. The results obtained agree with the fact, which I had discovered previously by a different method of analysis, that two non-congruent triple-systems may have the same group.