ON POINCARÉ'S CORRECTION TO BRUNS' THEOREM.

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THE differential equations of motion for the problem of three bodies were first set up by Clairaut, and were published by him with the remark, "Let anyone integrate them who can." Clairaut himself had found ten of the eighteen integrals necessary for the complete solution of the equations, but in despair gave up the hope of finding any more, contenting himself with methods of approximation for those cases which were presented by our solar system, particularly, the motion of the moon. The solutions of these equations have engaged the attention of nearly all of the great mathematicians from Clairaut down to the present time, but no more integrals have been forthcoming. This universal failure has given rise, naturally, to a suspicion that there are no more integrals of a simple type, and this suspicion has been strengthened by the researches of Bruns and of Poincaré. In 1887 Bruns published his famous theorem* that the equations of motion of the problem of n bodies ($n > 2$) do not admit any integral which is algebraic in the rectangular coördinates and in the time, other than the ten classical integrals which were found by Clairaut. Bruns' theorem was soon followed by another† due to Poincaré. According to Poincaré's theorem the equations of motion of the problem of n bodies ($n > 2$) do not admit any uniform transcendental integral for values of the masses sufficiently small, other than the ten classical integrals. Comparing his own theorem with that of Bruns, Poincaré has said:‡ "The theorem which precedes is more general, in a sense, than that of M. Bruns, since I have shown not only that there does not exist any algebraic integral but that there does not exist even a uniform transcendental integral, and not only that an integral cannot

* *Acta Mathematica*, vol. 11 (1887).
† *Acta Mathematica*, vol. 13.
‡ Les Méthodes nouvelles de la Mécanique céleste, vol. 1, p. 253.