γ being the angle between the directions θ, φ and α, β. By using the Mehler formulæ for Legendre's polynomials P_n, (18) may be transformed so as to contain elliptic sigma functions under a triple integral sign, giving a formula somewhat similar to (3).

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NOTE ON FERMAT'S LAST THEOREM.

BY PROFESSOR R. D. CARMICHAEL.

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THE object of this note is to prove the following

THEOREM. *If p is an odd prime and the equation*

$$x^p + y^p + z^p = 0 \tag{1}$$

has a solution in integers x, y, z each of which is prime to p, then there exists a positive integer s, less than $\frac{1}{2}(p-1)$, such that

$$(s+1)^{p^2} \equiv s^{p^2} + 1 \bmod p^3.$$

The proof is elementary. If there exists a set of integers x, y, z satisfying (1), there exists such a set having the further property that they are prime each to each. Consequently, for the purpose of argument we may assume that x, y, z have this property.

Then from elementary considerations it is known* that integers α, β, γ exist such that

$$x + y = \gamma^p, \quad y + z = \alpha^p, \quad z + x = \beta^p.$$

Therefore

$$(x+y)^{p-1} \equiv 1, \quad (y+z)^{p-1} \equiv 1, \quad (z+x)^{p-1} \equiv 1 \bmod p^2, \tag{2}$$

since $a^{p(p-1)} \equiv 1 \bmod p^2$ when a is prime to p.

From (1) it follows that

$$x + y + z \equiv 0 \bmod p,$$

* See, for instance, Bachmann's Niedere Zahlentheorie, Zweiter Teil, p. 467.