this quotient group. This implies that this quotient group is abelian and of type (2, 2) when $p = 2$, and when $p > 2$ it must contain at least $p$ invariant cyclic subgroups of order $p^2$. As this is contrary to the fact that $G$ contains $p + 1$ conjugate sets which involve generating operators of its maximal cyclic subgroups, we have proved that we arrive at an absurdity by assuming that $G$ does not involve any operator of order $p^{m-1}$, $m > 3$.

When $p > 2$ there are only two non-cyclic groups of order $p^m$ which involve operators of order $p^{m-1}$, $m > 3$, and each of these clearly contains maximal cyclic subgroups of order $p^\alpha$ which are transformed into themselves by more than $p^{\alpha+1}$ operators of $G$. Hence it results that the three non-cyclic groups of order $2^m$ which were considered above in the second paragraph are the only non-cyclic groups of order $p^m$ in which every maximal cyclic subgroup is transformed into itself by at most $p$ times as many operators of the group as there are operators in this maximal subgroup. This completes the proof of the theorem in question, and hence we can assume that *every non-cyclic group of order $p^m$, with the exception of the three of order $2^m$ which involve one and only one cyclic subgroup of order $2^{m-1}$, contains at least one maximal cyclic subgroup of order $p^\alpha$ which is transformed into itself by more than $p^{\alpha+1}$ operators of the group.*

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# AN EXPRESSION FOR THE GENERAL TERM OF A RECURRING SERIES.

BY PROFESSOR TSURUICHI HAYASHI.

PROFESSOR Arthur Ranum has given in the BULLETIN, volume 17, No. 9, June, 1911, pages 457–461, an explicit form of the general term of a recurring series rationally in terms of the first few terms and the constants of the scale of relation. I will give here another more explicit and more convenient form without demonstration.

Let $u_0 + u_1 + u_2 + \ldots + u_n + \ldots$ be any recurring series of order $n$, and let

$$u_m = a_1 u_{m-1} + a_2 u_{m-2} + \cdots + a_n u_{m-n} \qquad (m \geqq n)$$