

ITERATED LIMITS OF FUNCTIONS ON AN  
ABSTRACT RANGE.

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RESERVING details for a later publication, I wish here to indicate a method for the investigation of multiple and iterated limits under general conditions imposed on the range of the independent variable, and free from the unnecessary restrictions involved in the consideration of variables of particular types. For desirable simplicity of the general theory the postulates imposed on the range should not only permit of an effective definition of limiting element, but also be invariant under two fundamental processes :

(a) Extension of the system through the adjunction of ideal limiting elements.

(b) Composition of systems, e. g., the generation from two linear ranges of a two dimensional range.

In his thesis (Paris, 1906): "Sur quelques points du calcul fonctionnel" (*Rendiconti del Circolo Matematico di Palermo*, volume 22) M. Fréchet supposes the notion of limit of a sequence of elements to be defined for a certain abstract class and secures generalizations of several theorems on point sets and on continuous functions. Later, adopting an analogue of the distance relation (écart) between two points, he secures many more theorems. His postulates, however, do not readily admit of the introduction of ideal elements.

F. Riesz, in a paper before the International Congress of Mathematicians at Rome, 1908 ("Stetigkeitsbegriff und abstracte Mengenlehre," *Atti*, volume 2 (1908) pages 18-24) proposed a set of postulates in which the fundamental notion is that of a relation between subclass and element of the range, in the sense that the element is a "limiting" element of the subclass. By considering the properties of a system of subclasses that have a common limiting element he is able to define effectively an ideal limiting element as a system of subclasses having certain definite properties. The postulates of Riesz, however, are apparently better fitted for the treatment of abstract point set theory than for the treatment of functions on the range thus characterized.