

The work of M. Duhem is a monumental one and it is deserving of great commendation. He has made the learned world his debtor by this labor of love. He has not written a history of science, but he has composed a work of the kind that makes the history of science possible.

DAVID EUGENE SMITH.

*The Fundamental Theorems of the Differential Calculus.* By W. H. YOUNG. Cambridge University Press. ix + 72 pp.

As stated in the preface, "rigidity of proof and novelty of treatment have been aimed at rather than simplicity of presentation, though this has never been lightly sacrificed." The chapter headings are: I. Preliminary notions. II. Limits. III. Continuity and semicontinuity. IV. Differentiation. V. Indeterminate forms. VI. Maxima and minima. VII. The theorem of the mean. VIII. Partial differentiation and differentials. IX. Maxima and minima for more than one variable. X. Extensions of the theorem of the mean. XI. Implicit functions. XII. On the reversibility of the order of partial differentiation. XIII. Power series. XIV. Taylor's theorem.

The  $\epsilon$  argumentation usually found in such books is entirely absent. The notion of a limit point of a set of points is taken for granted and by means of it the whole theory of limits is constructed. Infinity is included among points approached as a limit point and hence without further particular statement functions are permitted to approach infinity as a limit the same as any other value. Following Baire, functions are considered as approaching multiple limits instead of one unique limit. Hence it comes about that many theorems which we are wont to see stated for certain classes of functions in terms of the equality of unique limits are here stated for more general classes of functions in terms of the equality or inequality of the upper or lower limits approached. As an example we select the following:

If, as  $x$  approaches the values  $a$ ,  $f(x)$  and  $F(x)$  have both the unique limit zero, or  $+\infty$ , or  $-\infty$ , then the limits of  $f(x)/F(x)$  lie between\* the upper and lower limits of  $f'(x)/F'(x)$ , provided

A.  $a$  is not a limiting point of common infinities of  $f'(x)$  and  $F'(x)$ ;

---

\*  $x$  is said to lie between  $a$  and  $b$  if  $a \leq x \leq b$ .