

## SHORTER NOTICES.

*Theories of Parallelism, an historical Critique.* By W. B. FRANKLAND. Cambridge, 1910. xviii + 70 pp.

AFTER reading *The Story of Euclid* (1901), and particularly after reading *Euclid Book I with a Commentary* (1902), by the same author, every student of the history of mathematics will welcome this little "historical critique," feeling sure of finding much food for thought in small compass. For it is one of the characteristics of Mr. Frankland that he says what he has to say in the fewest words possible, and hence in a book of less than a hundred pages he condenses matter that most writers would expand to fill double the space. And when we come to consider that in these few pages he has presented a scholarly digest of the theories of upward of forty geometers we begin to realize the thought that has been given to the subject and the skill that the author has shown.

Mr. Frankland gives a setting for the historical discussion in an introduction of eighteen pages. In this he begins by stating one existence assumption and one fundamental theorem as follows: "Let us assume that *straight lines* are freely applicable to themselves and to one another; and that there is a *plane* in which they are freely movable; and let us investigate the *parallelism* of such straight lines in an even plane." The theorem is that of Hilbert, that the area of any polygon is proportional to its divergence, that is, to the difference between its angle sum and  $(n - 2)\pi$ . From this theorem, of which the usual proof is given, the author proceeds to prove that for the elliptic, parabolic, and hyperbolic hypotheses, respectively, there are no, one, and two parallels to a given straight line, through any given point.

The historical sketch is then introduced by a discussion of Euclid's own theory, a discussion that has, of course, been anticipated by Dr. (now Sir Thomas) Heath's monumental work on the *Elements*. Heath calls attention to the fact that Euclid assumes the infinitude of space, and that the possibility of a straight line as re-entrant seems never to have occurred to him. He thus bars from his theory the possibility of the non-existence of parallelism. On the other hand the fifth postulate bars the possibility of double parallelism, so that there is left to him only the parabolic hypothesis. The weakest feature of his theory is that the statement of the fifth postulate positively invites attempts