and the conditions (β) and (γ) become, respectively,

$$(b) \quad \begin{vmatrix} F_\eta & F_{x_1} \\ G_\eta & G_{x_1} \end{vmatrix} = 0, \qquad (c) \quad \begin{vmatrix} 0 & F_\eta & F_{x_1} \\ F_\eta & F_{\eta z} & F_{zx_1} \\ G_\eta & G_{\eta z} & G_{zx_1} \end{vmatrix} \neq 0$$

for the system of values

$$x_1 = a_1, \quad x_2 = a_2, \cdots, x_n = a_n; \; \eta = 0, \quad z = c.$$

Hence the hypothesis of Theorem I is satisfied for the equations $F = 0$, $G = 0$, and we have a unique solution of the form

$$(20) \qquad \eta = \eta(x_1, x_2, \cdots, x_n), \quad z = \psi(x_1, x_2, \cdots, x_n),$$

such that

$$0 = \eta(a_1, a_2, \cdots, a_n), \quad c = \psi(a_1, a_2, \cdots, a_n).$$

Substituting the values of η and z in (19), we have the required solution for y.

If there is only one independent variable x, the conditions of Theorem II, for the system of values $x = y = z = 0$, imply that the equations have the form

$$f(x, y, z) \equiv [y - \lambda(z)] f_1(x, y, z) + x f_2(x, y, z) = 0,$$

$$g(x, y, z) \equiv [y - \lambda(z)] g_1(x, y, z) + x g_2(x, y, z) = 0.$$

The curve defined by these equations consists of two branches passing through the origin. One branch is the curve $y = \lambda(z)$ in the yz-plane and is not expressible by equations of the form $y = \phi(x)$, $z = \psi(x)$.

SHEFFIELD SCIENTIFIC SCHOOL,
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STURM'S METHOD OF INTEGRATING $dx/\sqrt{X} + dy/\sqrt{Y} = 0$.

BY PROFESSOR F. H. SAFFORD.

(Read before the American Mathematical Society, April 30, 1910.)

ONE of the simplest methods of obtaining the addition theorem for elliptic integrals of the first kind is based upon a method of integration which is usually referred to as Sturm's