GROUPS GENERATED BY TWO OPERATORS EACH OF WHICH IS TRANSFORMED INTO A POWER OF ITSELF BY THE SQUARE OF THE OTHER.

BY PROFESSOR G. A. MILLER.

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§ 1. *Introduction.*

Two special cases of the category of groups defined by the heading of this paper have been considered; viz., when the square of each of the two generators transforms the other generator either into itself* or into its inverse.† It was observed that in the former of these two cases the orders of the two generators are not restricted, while in the latter each of these orders must divide 8. Each of these special cases led to a very elementary category of solvable groups. It will be proved that the more general category defined by the heading of this paper is also composed entirely of solvable groups of simple structure.

As an instance of how such generalizations may lead to very complex categories of groups we may give the theorem that *every symmetric group can be generated by two operators whose squares are commutative.* In fact, the symmetric group of degree n is evidently generated by the following two cyclic substitutions whose squares are commutative:

$$t_1 = (x_1 x_2 x_3 \cdots x_{n-1}), \quad t_2 = (x_1 x_n).$$

From the theorem that every symmetric group whose degree exceeds 8 can be generated by two substitutions of orders 2 and 3 respectively ‡ it results directly that all such groups are included in the category of groups defined by the condition that each of them can be generated by two operators which are transformed into themselves by the square and cube respectively of the other.

* BULLETIN, vol. 16 (1910), p. 173.

‡ *Annals of Mathematics*, vol. 9 (1907), p. 48.

‡ BULLETIN, vol. 7 (1901), p. 426.