ON SOME THEOREMS IN THE LIE THEORY.

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THE well-known treatment of the infinitesimal transformation in the Lie theory makes use of power series, assuming that the functions involved are analytic. The treatment here given demands at most the existence of second partial derivatives. Since no use is made of the group theory, the proofs here given may be used either in connection with the group theory or apart from it.

Three theorems are stated in this paper, of which Theorem I. was proved in a previous paper.* Of the three, any one is an immediate result of the other two taken together. They may be compactly stated as follows:

Given any differential equation of the form

$$\Omega\,(x, y, y') = 0 \tag{1}$$

which, in a given region R, can be written in the form linear in y'

$$\Omega_1 \equiv X(x, y)y' - Y(x, y) = 0, \tag{2}$$

and of which $\omega(x, y) = c$ is an integral; let $\xi(x, y)$ and $\eta(x, y)$ be such functions that

$$X\eta - Y\xi \neq 0.$$

Consider the statements

A. *The equation*

$$\frac{X}{X\eta - Y\xi}y' - \frac{Y}{X\eta - Y\xi} = 0 \tag{3}$$

is exact. That is, $1/(X\eta - Y\xi)$ is an integrating factor of (2).

B.

$$\xi\frac{\partial\Omega}{\partial x} + \eta\frac{\partial\Omega}{\partial y} + \eta'\frac{\partial\Omega}{\partial y'} = 0,\dagger \tag{4}$$

* BULLETIN, vol. 15, no. 8 (May, 1909).

† In the Lie theory this is the condition for invariance under the extended transformation

$$U'f = \xi\frac{\partial f}{\partial x} + \eta\frac{\partial f}{\partial y} + \eta'\frac{\partial f}{\partial y'}.$$