A NEW PROOF OF WEIERSTRASS'S THEOREM CONCERNING THE FACTORIZATION OF A POWER SERIES.

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THE theorem which is to be proved here may be stated in the following form:

Let $f(x_1, x_2, \cdots, x_p, y)$ *be a convergent series in* $x_1, x_2, \cdots, x_p, y$, *and such that the series* $f(0, 0, \cdots, 0, y)$ *begins with the term of degree* n. *Then* $f(x_1, x_2, \cdots, x_p, y)$ *is factorable in the form*

$$f(x_1, x_2, \cdots, x_p, y) = (a_0 + a_1 y + a_2 y^2 + \cdots + a_{n-1} y^{n-1} + y^n)\phi(x_1, x_2, \cdots, x_p, y),$$

where $a_0, a_1, \cdots, a_{n-1}$ *are convergent power series in* $x_1, x_2, \cdots, x_p$ *which vanish for* $x_1 = x_2 = \cdots = x_p = 0$, *and* ϕ *is a power series in* $x_1, x_2, \cdots, x_p, y$ *which has a constant term different from zero.*

In the *Bulletin de la Société Mathématique de France* * Goursat has called attention to the fact that the proof which Weierstrass gave of this important theorem, as well as the later proofs which occur in the literature † make use of the notions of the function theory, while the theorem itself is essentially of an algebraic character. In the paper referred to he has given an elegant and elementary proof of the theorem which is in outline as follows:

By means of the substitution

$$y^n = -a_0 - a_1 y - a_2 y^2 - \cdots - a_{n-1} y^{n-1}$$

the series f can be reduced to a polynomial P of degree $n-1$ in y, whose n coefficients are convergent series in $a_0, a_1, \cdots, a_{n-1}, x_1, x_2, \cdots, x_p$. By the usual theorems in implicit function theory it is shown that the n equations found by putting these coefficients equal to zero have unique solutions for $a_0, a_1, \cdots, a_{n-1}$ as power series in $x_1, x_2, \cdots, x_p$ which vanish with $x_1, x_2,$

* "Démonstration élémentaire d'un théorème de Weierstrass," vol. 36 (1908), p. 209.

† Picard, Traité d'Analyse, vol. II, p. 243; Goursat, Cours d'Analyse, vol. II, p. 284.