For the cycloid

$$\frac{1}{\rho^2}=\frac{1}{16a^2\sin^2\frac{1}{2}\theta}=\frac{1}{8a\left(s-\dfrac{s^2}{8a}\right)}=\frac{1}{8ay},$$

and after substitution of these values in $\overline{K}_0$ we find

$$\overline{K}_0=\frac{1}{8ay}\{2a(y'^2-3)+y\},$$

or making use of the extremal equation,

$$\overline{K}_0=-\frac{1}{2y}, \quad i.\ e., \quad \overline{K}_0<0.$$

By means of theorem A, we have the result:

For the brachistochrone problem there is no conjugate point to any point P lying on the same cycloid arch with P.

THE UNIVERSITY OF WISCONSIN.

A SIMPLER PROOF OF LIE'S THEOREM FOR ORDINARY DIFFERENTIAL EQUATIONS.

BY PROFESSOR L. D. AMES.

(Read before the Chicago Section of the American Mathematical Society, April 9, 1909.)

THE following theorem is essentially equivalent to Lie's principal theorem concerning the integration of the differential equation $\Omega(x, y, y')=0$ when it is invariant under a known group. As stated here, this theorem makes no use of the idea of a group.

THEOREM. *Given any differential equation of the form*

$$(1) \qquad \Omega(x, y, y')=0$$

which can be solved in the form

$$(2) \qquad X(x, y)y'-Y(x, y)=0;$$

if $\xi(x, y)$ and $\eta(x, y)$ are such functions that

$$X\eta-Y\xi \not\equiv 0,$$